Dynamic synchrophasor estimation by Taylor–Prony method in harmonic and non-harmonic conditions

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Abstract: Dynamic phasor is suitable for analysis of non-stationary signals (such as power swings) in phasor measurement unit. In this study, dynamic phasor of the generated signals during power swing is analysed by a classical signal processing technique known as Prony. The Prony analysis is a useful technique to model a linear sum of damped complex exponential signals. In this study, a combination of the least square-based Prony analysis and Taylor expansion called as Taylor–Prony is proposed to estimate the dynamic phasor. Zer0th-order Taylor–Prony, i.e. static phasor is compared with the second-order Taylor–Prony, i.e. dynamic phasor for illustrating the dynamic phasor capability. Sinusoidal and step changes of amplitude and phase, harmonic condition, frequency tracking tests, noise infiltration and computation time are different tests which are used to validate the proposed method. First, the method is explained theoretically, and then its potential is demonstrated by simulating various numerical signals in MATLAB and measured signals during power swing from three-machine power system simulated in PSCAD. Simulation results show that the proposed method can estimate the dynamic phasor during power swing accurately. Ultimately, the proposed method is compared with six methods which have already presented to verify the capability of the proposed method.

1 Introduction

Dynamic phasor is a complex envelope of sinusoidal signals with variable amplitude and phase. This concept plays an important role in the analysis of power systems during quasi-steady state conditions. Power systems are constantly subjected to disturbances and any disturbance may move them to stressed conditions such as power swing. The power swing is a phenomenon, creates large fluctuations of active and reactive powers between two areas of an electrical system resulted from severe disturbances such as line faults, loss of generator units and switching heavy loads [1, 2].

Various researchers have suggested different analytical methods to estimate dynamic phasor. In [3], a new method based on adaptive band-pass filter has been proposed to estimate phasor. Jin et al. [4] has introduced an angle-shifted energy operator in order to extract the instantaneous amplitude. An integrated phasor and frequency estimation using a fast recursive Gauss–Newton algorithm has been proposed in [5] and a method based on modified Fourier transform for eliminating direct current (DC) offset has been presented in [6]. A phasor estimation algorithm based on the least square curve fitting has been proposed in [7] to overcome current transformer saturation effect. In [8], an approach for estimating the phasor parameters based on recursive wavelet transform has been introduced. Phadke et al. [9] has discussed phasor and frequency measurements under transient system conditions. Maximally flat differentiators [10] and phasorlet [11] are other methods for dynamic phasor estimation. Taylor Fourier algorithm which has been proposed in [12], approximates the dynamic phasor by second-order Taylor expansion and least square observer. Taylor Kalman method uses Kalman observer to estimate dynamic phasor without delay [13] and has been enhanced by developing state space for harmonic infiltration in [14], named Taylor Fourier Kalman. Shank method is another dynamic phasor estimation method, which employs least square and consecutive delays of unit response system [15]. In [16], Prony algorithm has been used to estimate phasor with exponential amplitude and linear phase. However, Taylor expansion is not used in this method. There are also other methods which are modification of earlier methods [17, 18].

The Prony algorithm approximates the main signal by exponentially damped sinusoidal signals. This algorithm is able to determine the values of frequency, damping factor, amplitude and phase of the main signal. By this algorithm, frequency and damping factor parameters are calculated in first step and consequently amplitude and phase parameters are obtained in second step [16]. Prony tool has many applications in different areas such as power system frequency tracking, fault inception detecting, power swing detecting, modal analysing of low-frequency oscillation and improving over current relay operation.

In this paper, a combination of Taylor expansion with least square Prony method is used to estimate the dynamic phasor, named Taylor–Prony. Accuracy of zeroth-order Taylor–Prony (static phasor) is compared with second-order Taylor Prony (dynamic phasor) by using total vector error (TVE), phasor estimation error (PEE), harmonic situation, frequency tracking and noise infiltration tests. Then the proposed method is compared with six phasor estimation methods, which have already presented. The method is described theoretically first and consequently validated by test signal simulated in MATLAB and three-machine power system, simulated in PSCAD.

2 Dynamic phasor estimation based on Prony method

Phasor estimation process by Taylor–Prony algorithm for both non-harmonic (first part) and harmonic condition (second part) are presented in this section.

2.1 Phasor estimation in non-harmonic condition

Consider a damped sinusoidal signal as:

\[ y(t) = Ae^{-\alpha t} \cos(\omega t + \phi) \]  

(1)

where \( A, \alpha, \phi \) and \( \omega \) are amplitude, damping coefficient, phase and angular frequency of fundamental component, respectively. Using Euler’s theorem we have:
\[ y(t) = A(t)e^{j\omega t} + A(t)e^{-j\omega t} \] (2)

Assume that the signal \( y(t) \) is sampled at \( N \) samples per one fundamental cycle \((t = nT)\). Therefore, in discrete condition we have \( Z_n = e^{j\omega t} \) and so:

\[ y(n) = HZ_n^T + H'Z_n^n \] (3)

where \( T \) is sampling period and \( H = A(t)e^{j\beta(t)} \) is static phasor. This is actually the phasor of sinusoidal signal with constant amplitude and phase. However, this condition (constant amplitude and phase) is a limitation for phasor estimation during power swing because amplitude and phase of main signal are time-variant during power swing. Hence, to reach higher accuracy in phasor estimation, the amplitude and phase should be considered as time variables. Therefore, (2) is replaced with (4) as:

\[ y(t) = A(t)e^{j\omega t} + A(t)e^{-j\omega t} \] (4)

According to (4), \( H(t) = A(t)e^{j\beta(t)} \) is comprehensive definition of phasor as dynamic phasor with time-varying amplitude and phase. Regarding time-varying character of dynamic phasor, its second-order Taylor expansion around the interval centre \((t = 0)\) is calculated as:

\[ H(t) = h_0 + h_1 t + h_2 t^2 \]
\[ h_0 = H(t = 0) \]
\[ h_1 = \frac{dH}{dt}(t = 0) \]
\[ h_2 = \frac{1}{2} \frac{d^2H}{dt^2}(t = 0) \] (5)

where \( h_0, h_1 \) and \( h_2 \) are mean value, first derivative and second derivative of dynamic phasor. According to (4), (5) we have:

\[ y(t) = (h_0 + h_1 t + h_2 t^2)e^{j\omega t} + (h'_0 + h'_1 t + h'_2 t^2)e^{-j\omega t} \] (6)

Assume that the main signal (6) is sampled at \( N \) samples \((n = 0, \ldots, N - 1 = N')\) per one fundamental cycle and expressed in (7) as abbreviated form (detailed form of this equation is presented at the bottom of page \((Z_n = e^{j\omega t})\)). (see equation below)

\[ Y = Jh \] (7)

The second order estimation of \( h \) is obtained by least square method as:

\[ h = (J^h J)^{-1} J^h Y \] (8)

where \( h \) is Hermitian operator. By determining \( Z_n \), the dynamic phasor coefficients \((h_0, h_1, h_2, h'_0, h'_1, h'_2)\) can be calculated based on (8) which represents a method which its output depends on present and past inputs. According to (8), in order to calculate coefficients of dynamic phasor, it is necessary to calculate \( Z_n \) first. To calculate \( Z_n \), it is necessary to formulate characteristic equation as:

\[ F(z) = (z - Z_1)(z - Z_2) = a_0z^2 + a_1z + a_2 = \sum_{m=0}^{L} a_mz^{-m} \] (9)

where, \( L = 2 \). According to (9), in order to solve the characteristic equation, it is necessary to calculated \( a_0, a_1 \), and \( a_2 \) (characteristic equation coefficients) first. Based on (3), by shifting the index from \( n \) to \( n - m \) and multiplying by \( a_m \) (10) is resulted as:

\[ a_m[n - m] = \sum_{k=0}^{L} H_k z^{-m} \] (10)

By breaking the right-hand side of (10) in two summations, (11) is obtained as:

\[ \sum_{m=0}^{n} a_m[n - m] = \left( \sum_{k=0}^{L} H_k z^{-m} \right) \left( \sum_{m=0}^{L} z^{-m} \right) \] (11)

Based on (9), \( Z_n \) and \( Z_n' = Z_n' \) are the roots of characteristic equation \((\sum_{m=0}^{L} a_mz^{-m} = 0)\), hence the right-hand side of (11) equals to zero. Then:

\[ \sum_{m=0}^{n-p} a_m[n - m] = 0 \] (12)

If the equation of (12) is extracted and formulated for \( N \) samples, the outputs can be expressed as matrix form:

\[
\begin{bmatrix}
  y[0] \\
  y[1] \\
  \vdots \\
  y[N-1]
\end{bmatrix} =
\begin{bmatrix}
  \vdots & \vdots & \vdots & \ddots & \vdots \\
\end{bmatrix}
\begin{bmatrix}
  a_0 \\
  a_1 \\
  a_2
\end{bmatrix}
\] (13)

where \( r \) is transpose operator. Therefore, dynamic phasor estimation based on Taylor–Prony method is divided into three steps. The first step involves calculating the coefficients of characteristic \((a)\) using \( N \) samples point of \( y[n] \) based on (13). By substituting the amounts of the coefficients \((a)\) in (9), \( Z_n \) and \( Z_n' \) can be obtained as its roots in the second step. The final step is to solve (7) in order to obtain coefficients of dynamic phasor. The flowcharts of proposed algorithm for dynamic phasor estimation is shown in Fig. 1.

2.2 Phasor estimation in harmonic condition

Taylor–Prony method proposed in previous section operates successfully when input signal relies on the model which it is designed on. Proposed method in previous section is designed based on (6) so it just operates appropriately when subjected to fundamental frequency input \((\omega = \omega_0)\). Complete model of input signal is necessary to guarantee proper operation of mentioned
method in contaminated power system by harmonics. Therefore, new model of main signal is considered as:

\[
\begin{align*}
y &= H^{(0)}z_0^n + (H^{(1)})z_1^n + \\
H^{(1)}z_1^n + (H^{(2)})z_2^n + \\
H^{(2)}z_2^n + (H^{(3)})z_3^n + \\
&\cdots + \\
H^{(N-1)}z_{N-1}^n + (H^{(N-1)})z_N^n,
\end{align*}
\]

where \( n \) is sample number (\( n \in \{0, 1, 2, \ldots, N-1\} \)), \( N \) is sample number per one fundamental cycle and \( z_0, z_1, z_2, \ldots, z_{N-1} \) are roots of characteristic equations for zero harmonic (DC component), first harmonic (fundamental component), second harmonic, \ldots, and \((N-1)\)th harmonic respectively. Further, \( H^{(0)} = \alpha A \) is dynamic phasor of DC component, \( H^{(1)} = \beta A \) is dynamic phasor of fundamental component, \( H^{(2)} = \gamma A \) is dynamic phasor of second harmonic component, \ldots, and \( H^{(N-1)} = \phi A \) is dynamic phasor of \((N-1)\)th harmonic component. Based on complete model presented in (15), second-order Taylor expansion of dynamic phasor of harmonics around \( t = 0 \) are:

\[
\begin{align*}
H^{(0)}(t) &= h^{(0)} + h^{(0)}t + h^{(0)}t^2 \\
H^{(1)}(t) &= h^{(1)} + h^{(1)}t + h^{(1)}t^2 \\
&\cdots \\
H^{(N-1)}(t) &= h^{(N-1)} + h^{(N-1)}t + h^{(N-1)}t^2.
\end{align*}
\]

Based on this model, (7) is converted to (17) and (13)–(18).

\[
\begin{bmatrix}
y[0] \\
y[1] \\
\vdots \\
y[N-1]
\end{bmatrix} = \begin{bmatrix}
y[0] \\
y[1] \\
\vdots \\
y[N-1]
\end{bmatrix} = J^{[0]}J^{[1]} \cdots J^{[N-1]} \begin{bmatrix}
h^{(0)} \\
h^{(1)} \\
\vdots \\
h^{(N-1)}
\end{bmatrix}
\]

where \( J^{[0]}, J^{[1]}, \ldots, J^{[N-1]} \) are harmonic descriptions of matrix \( J \) explained in (7); \( h^{(0)}, h^{(1)}, \ldots, h^{(N-1)} \) are harmonic descriptions of matrix \( h \) explained in (7); and \( a^{[0]}, a^{[1]}, a^{[N-1]} \) are harmonic descriptions of matrix \( a \), explained in (13). According to (17) and (18), it is possible to estimate dynamic phasors of first \( N-1 \) harmonics (DC, fundamental, second harmonic, \ldots, \((N-1)\)th harmonic). To sum up, according to mentioned formulations of this section, it is possible to extract dynamic phasor of every harmonic individually and then estimation of fundamental dynamic phasor in harmonic condition accurately.

3 Simulation results

Two general simulation parts are provided in this section. The first part is to the assessment of proposed method performance for synthesised signal in MATLAB. The second part is a simulation of three-machine power system in PSCAD and analysis of measured signal in MATLAB. Since proposed method can be used in phasor measurement unit (PMU) and current PMU standard provides a series of tests by which PMU should be conformed, some of these tests are considered in this paper and performance of proposed method is analysed by these tests.

3.1 Simulation results for synthesised signal in MATLAB

The main signal which is used to validate the proposed method is the main signal which is used to validate the proposed method is

\[
y(t) = a(t)\cos(2\pi f_1 t + \phi(t))
\]

where

\[
a(t) = a_0 + (a_1 \sin(2\pi f_1 t))e^{-t^2} \\
\phi(t) = \phi_0 + (\phi_1 \cos(2\pi f_1 t))e^{-t^2} \\
a_0 = 1, \phi_0 = 0.5, a_1 = 0.1, \phi_1 = 0.05 \\
f_1 = f_0 = 5, N = 20, r = 0.5
\]

The fundamental and sampling frequencies are 50 Hz and 1 kHz, respectively. Moreover, time window is equal to one fundamental cycle hence there are 20 samples \((N=20)\) in time window. It is worth noting that by utilisation of Prony method, it is possible to extract damping factor, frequency, amplitude and phase angle. Since the purpose of this paper is to track phasor (amplitude and phase) and frequency, we have considered zero value for damping factor \((\alpha = 0)\) in (19). Therefore, (19) is resulted from (4) just by zero consideration of damping factor.

3.1.1 Estimation of dynamic phasor (amplitude and phase): Fig. 2a shows the amplitude estimation of dynamic phasor (entitled as dynamic phasor Prony) and static phasor (entitled static phasor Prony). Fig. 2b shows the phase estimation of dynamic and static phasors as well. In these figures, the ideal amplitude and phase are shown in dashed, while their estimates are in solid lines. As the first result of this part, both static phasor Prony and dynamic phasor Prony methods can track real amplitude and phase correctly by one cycle delay. Distortion in output of static phasor Prony is perceptible because the speed (first derivative of phasor) and acceleration (second derivative of phasor) components have not been considered in estimation process. Hence the estimation of dynamic phasor Prony is more precise than static phasor. In order to show phasor estimation capability of dynamic phasor Prony over to static phasor Prony method more clearly, complex trajectory of phasor estimations are illustrated in complex plane as shown in
These results present the higher accuracy of dynamic phasor compared with static one as a noticeable advantage. It is worth noting that there are some fluctuations in TVE values (Fig. 4) which are due to oscillating amplitude of main signal during power swing.

3.1.2 Derivatives of dynamic phasor: To extract first derivative of amplitude and phase, consider first derivative of dynamic phasor, $H(t) = A(t) \times e^{j\phi(t)}$, as:

$$H'(t) = A'(t) \cdot e^{j\phi(t)} + j\phi'(t) \cdot A(t) \cdot e^{j\phi(t)} \quad (21)$$

where $A'(t)$ and $\phi'(t)$ are first derivative of amplitude and phase, respectively. By employing $e^{-j\phi(t)}$ to both sides of (21) and separating real and imaginary parts we have:

$$A'(t) = \text{real}[H'(t) \cdot e^{-j\phi(t)}] \quad (22)$$

To extract second derivative of amplitude and phase, consider the second derivative of dynamic phasor as:

$$H''(t) = A''(t) \cdot e^{j\phi(t)} + 2j\phi''(t) \cdot A(t) \cdot e^{j\phi(t)} + j\phi'(t) \cdot A'(t) \cdot e^{j\phi(t)} - \phi'(t)^2 \cdot A(t) \cdot e^{j\phi(t)} \quad (24)$$

where $A''(t)$ and $\phi''(t)$ are second derivative of amplitude and phase, respectively. By employing $e^{-j\phi(t)}$ to both sides of (24) and separating real and imaginary parts we have:

$$A''(t) = \text{real}[H''(t) \cdot e^{-j\phi(t)} + \phi''(t)^2 \cdot A(t)] \quad (25)$$

$$\phi''(t) = \frac{\text{imag}[H''(t) \cdot e^{-j\phi(t)}] − 2\phi'(t) \cdot A'(t)}{A(t)} \quad (26)$$

It is worth noting that the dynamic phasor is considered $H(t) = A(t)e^{j\phi(t)}$ in this paper. According to definition of dynamic phasor, the amplitude and phase can be any time variant quantity likes as $A(t) = A_0 + (A_1 \cos(\omega t) + \epsilon^{-i\omega t})$ which is considered in simulation sections. In (21)–(26), we have considered general time variant quantity for dynamic phasor which can be any pattern such as attenuated exponential. Figs. 5a and b represent first and second derivatives of amplitude of main signal. According to the figures, dynamic phasor is able to calculate phasor's derivatives but static phasor does not have this ability. The real and estimated derivatives are shown in dashed and solid lines, respectively. It is worth noting that the first and second derivatives estimations of phasor (Figs. 5a and b) are not as accurate as the amplitude estimation (Fig. 2a) due to elimination of third derivative in Taylor Expansion.

Phasor's derivatives play two important roles. First, they reduce error estimations, as shown in simulation results; and second, they...
are able to detect changes like faults and power swings. Many schemes have been proposed in order to detect fault, power swing and three-phase fault during power swing in literatures [19, 20, 21]. According to the above mentioned explanations, first and second derivative of dynamic phasor are functional to be used in this field as its superiority compared with static phasor.

### 3.1.3 Magnitude-phase step test

To investigate the dynamic response of proposed method, dynamic benchmark based on amplitude-phase step is considered in this section. The test case has the form as:

\[
y(n) = a(n)\cos(2\pi f_n n \Delta t + \phi(n))
\]

where:

- \(a(n) = 1.0, \phi(n) = 00 < n < 10N_1\)
- \(a(n) = 1.5, \phi(n) = \frac{\pi}{4}, n = 10N_1\)  \hspace{1em} (27)
- \(a(n) = 2, \phi(n) = \frac{\pi}{2}, 10N_1 \leq n\)

It is 100% magnitude step and 90° phase step. The simulation results of this test case are shown in Fig. 6. According to Fig. 6, the estimated amplitude and phase track their real values accurately after transient period. This transient period is related to data window which is one fundamental cycle. The reason of this transient response comes from Taylor model which is more appropriate for smooth signals and not sudden changes in signals.

### 3.1.4 Frequency tracking test

The test waveform of this section has been considered as 5 Hz frequency step and is presented in (28) as:

\[
y(n) = \cos(2\pi f_n n \Delta t) \quad 0 < n < 10N_1
\]
\[
y(n) = \cos(2\pi (f_n + 5) n \Delta t) \quad 10N_1 \leq n
\]

Although 5 Hz frequency step is not likely to happen in a power grid, this test is designed based on IEEE standard (IEEE Std. C37.118 [1]). This test can evaluate one of the main contributions of Prony analysis. Frequency of input signal to Prony method is calculated as:

\[
\text{frequency} = \frac{\text{imag(root}[a_0, a_1, a_2])}{2\pi}
\]

where root calculates roots of characteristic equation based on extracted coefficients \((a_0, a_1, a_2)\) from (13). Fig. 7 demonstrates the response of frequency estimation of Prony method when subjected to frequency step. According to the figure, +5 Hz frequency step is tracked by both static and dynamic phasor Prony methods in a short period (one cycle) without ripple. It is worth noting that the frequency is calculated based on (13) which are similar for both static and dynamic phasors. Therefore, the outputs of both concepts for frequency tracking test are exactly equal.

Power system frequency measurement has been used since the introducing of AC generators and systems. Several techniques for measuring power system frequency have been published in technical literature as [22, 23, 24]. The estimation of power signal frequency has numerous applications in control and protection areas of power systems. Furthermore, penetration of distribution resources in electrical networks is absolutely dependent on accurate estimation of power system parameters such as frequency. Thus frequency measurement is important in power system operation. The frequency and its change rate are commonly used as indicators for restoration and islanding of power systems. Prony analysis can offer an opportunity for measuring power system frequency as shown in Fig. 7. It can be observed that both static and dynamic phasor Prony methods have frequency tracking ability as its intrinsic property.

### 3.1.5 Error bounds

Test signal, presented in (30), with variable envelope is utilised to compare the error range of static and dynamic phasor Prony methods. Hence, 200 signals with fluctuating amplitude were generated over 25 cycles as:

\[
y(n) = a(n)\cos(2\pi f_n n \Delta t + \phi(n))
\]
where the time constants ($\tau$) are generated by an uniform random process in the interval of [20–40] cycles. In a similar way, the three frequencies ($f_i$) are randomly generated in [1–5], [3–5] and [5–7] Hz intervals. Therefore, the estimation error is calculated by:

$$\text{rms}\{\text{error}\} = \sqrt{\sum_{i=1}^{N} (a_i(t) - \hat{a}_i(t))^2}$$

where $\hat{a}_i$ is estimation of $i$th signal ($a(t)$) by Prony method. The minimum and maximum values of the estimation error can be determined by histogram tool as well as the most probable error.

Fig. 8 shows the histograms of estimation errors of static and dynamic phasor Prony. The RMS error belongs to the range of $3 \times 10^{-3}$ and $8 \times 10^{-3}$ for dynamic phasor Prony method. This error is related to elimination of third derivative term of Taylor Expansion in phasor estimation process. The RMS error value is also placed between $3 \times 10^{-2}$ and $8 \times 10^{-2}$ for static phasor Prony method. This test can demonstrate the superiority of dynamic phasor (second-order Taylor) over than static phasor (zeroth-order Taylor) in estimation accuracy.

$\textbf{3.1.6 Harmonic infiltration test}: \text{Main signal (32) has been considered in this section to validate the proposed methods in harmonic condition. The main signal consists of three components; 50 Hz (100%) component (fundamental), 250 Hz (5%) components (fifth harmonic) and 350 Hz (3%) components (seventh harmonic)}$

$$y(t) = a(t)\cos(\omega_0 t + \phi(t)) + 0.05\cos(5\omega_0 t) + 0.03\cos(7\omega_0 t)$$

where

$$a(t) = a_0 + (a_1\sin(2\pi f_0 t))$$

$$\phi(t) = \phi_0 + (\phi_1\cos(2\pi \cdot f_0 t))$$

$$a_0 = 1, \phi_0 = 0.5, a_1 = 0.1, \phi_1 = 0.05$$

$$f_0 = f_\phi = 5 \text{ Hz}$$

Fig. 9 represents the output of dynamic phasor Prony method in harmonic condition (ideal and estimated amplitudes have been shown in dashed and solid lines, respectively). According to Fig. 9, fundamental phasor estimation based on modified model is free from harmonic. Therefore, this test shows the superiority of harmonic modification of proposed method in this paper. Since $N = 20$ in this test, it is possible to estimate dynamic phasor of first $N - 1 = 19$ harmonics. To increase range of harmonics, the sampling number per cycle should be increased. For example by considering $N = 50$, it is possible to estimate up to 49th order of harmonic.

$\textbf{3.2 Simulation results for the three-machine power system}$

In order to demonstrate the capability of the proposed technique for signals obtained from power system, three-machine power system has been considered as shown in Fig. 10 (the data of this power system are presented in [19]). Power system has been simulated in PSCAD software. In this study, a distance relay is located in line between bus-8 and bus-9. In order to create a power swing, a three-phase fault is created in the line between buses 6 and 9. The fault is started at $t = 1 \text{s}$ and cleared after 0.2 s by opening circuit breakers located at ends of this line which causes a power swing in line between buses 8 and 9 and is observed by the distance relay $R$.

The power system is simulated by sampling frequency 10 kHz in PSCAD. In order to model distance relay, sampling rate of measured signal is downsampled by integer factor 10 in MATLAB and is sent to proposed method to estimate its phasor. Moreover, time window is equal to one fundamental cycle so there are 20 samples ($N = 20$) in time window. Phase-a current waveform, observed by the relay $R$ is considered for analysis in this section. The proposed method (dynamic phasor Prony) has been compared with six methods which have already presented in literatures as:

- Method1: This method is based on second-order Taylor Expansion and Fourier transformation, and uses least square observer to approximate dynamic phasor [12].
- Method2: This method is based on second-order Taylor Expansion and Kalman filter to approximate dynamic phasor instantaneously to address delay challenge [13].
- Method3: The contribution of this method is utilisation of a special digital filter design (named shanks) which employs least square method and consecutive delays of unit response to estimate dynamic phasor [15].
- Method4: This method [Phase lock loop (PLL) Taylor Fourier] is proposed in [25]. The main idea of this paper is to estimate the phasor in each window using the subspace defined by the polynomial phase obtained in the precedent observation window. This improves the dynamic phasor estimates obtained.
always since the static subspace defined by the linear phase corresponding to the ideal fundamental frequency. This algorithm is permanently adapting to the phase of the last given signal segment, or the nearest given phase Taylor polynomial.

- **Method 5**: This method (Enhanced PLL) is proposed in [26]. This paper introduces a continuous adjustment non-window approach based on Enhanced Phase locked loop (EPLL) for estimating synchrophasor within off-nominal frequency operation of the system. Since the major concern in PMU is accurate estimation within off-nominal frequency conditions, this paper proposes a new an effective tool for phasor measurement in frequency varying condition.

- **Method 6**: This method (Prony method) is proposed in [16]. This paper proposes an approach of dynamic phasor estimation by utilisation of pre-estimation property of Prony method. In this method, just the constant term of Taylor series of phasor is used in Prony analysis. It is worth noting that our proposed method actually extrapolates the method proposed in [16].

### 3.2.1 Accuracy of phasor estimation

Six mentioned methods and proposed method are employed to estimate dynamic phasor of current obtained from three-machine power system and the results are provided in Fig. 11. Regarding this figure, all methods can track amplitude correctly. In order to compare the accuracy of all methods, the index of PEE is used. Since the real value of phasor is not available in this kinds of tests, it is not possible to use TVE in this section. Index of PEE is defined as:

\[
\text{PEE}(n) = \left| y(n) - \hat{y}(n) \right|
\]

\[
\hat{y}(n) = |H(n)| \cos(\theta n + \angle H(n))
\]

where \( n \) is sample number, \( \theta = 2\pi/N \), \( N = 20 \) is sample number per fundamental cycle, \( y(n) \) is measured signal and \( \hat{y}(n) \) is recomputed sample based on estimated dynamic phasor \( (H(n)) \). Mean value of PEE of mentioned six methods and proposed method are tabulated in Table 1. According to this table, proposed method provides the lowest mean value of PEE compared with other six methods. This low value of error validates the high accuracy of proposed method. It is worth noting that the accuracy of **Method 4** is also as well as proposed method.

### 3.2.2 Computation time

Computation time is another critical index for comparison of different phasor estimation methods. Three-machine power system is simulated by Intel(R) Core(TM) 2 Duo CPU (T9550) processor and computation time of all seven methods are tabulated in Table 1. According to this table, the proposed method has the CPU time about 0.61 s to calculate 5000 phasor. **Method 5** is the fastest one (about 0.41 s), and **Method 3** is the slowest algorithm (about 1.23 s) in this comparison.

### 3.2.3 Noise infiltration

It is useful to analyse the estimation error when noise level changes. Mean value of PEE is used in this section which is calculated by difference of measured sample and recomputed sample obtained by estimated dynamic phasor. Fig. 12 shows this index value as a function of noise variance for seven methods. As the noise variance increases, higher errors are obtained by all methods. So there is an upward trend when noise...
level increases. According to Fig. 12, there is a stable trend for Method1 before critical point with variance about 10^-3. After this point, PEE increases steeply by increasing error variance. Moreover, it can be seen that the proposed method has the best performance when noise variance is low. However, when noise variance becomes bigger than 10^-3, Method1 is the best method (lowest error). Therefore, this test shows the limitation of proposed method in high value of noise variance.

4 Conclusion

Dynamic phasor concept is used in this paper to estimate variable amplitude and phase more precisely than static phasor estimation methods. Taylor–Prony method is proposed in this paper to estimate dynamic phasor in non-harmonic and harmonic conditions. Static phasor Prony is compared with dynamic phasor Prony to show the accuracy of the proposed method. TVE, PEE, harmonic condition, frequency tracking and noise infiltration tests are used to show the capability of proposed method. Moreover, six present methods are compared with the proposed method in this paper. Simulation results demonstrate that the Taylor–Prony method gives accurate results (amplitude, phase and frequency) during power swing in power systems.

5 References


Fig. 12 Mean of PEE of three existing methods and proposed method (Taylor Prony) for three-machine power system